**Big-O:**

∃c>0.∃n0≥1.∀n≥n0.f(n)≤cg(n)

**Big-Omega:**

∃c>0.∃n0≥1.∀n≥n0.f(n)>=cg(n)

**Theta:**

∃c’>0, ∃c’’>0, .∃n0≥1.∀n≥n0. c’g(n)<=f(n)<=c’’g(n)

**Limit Asympthotic Theorem:**

lim |f(n)/g(n)| = L; n tends to infinity.

If L = 0, then f(n) is O(g(n)).

If L = C, then f(n) is Theta(g(n)).

If L = infinite, then f(n) is Omega(g(n)).

L’Hospital rule:

(n tends to infinity)

If lim f(n) = infinite and lim g(n) = infinite,

then lim (f(n)/g(n)) = lim(f’(n)/g’(n)).

**Big-O:**

**Ex 1:**

f(n) = n + 5

g(n) = n

Show that f(n) is O(g(n))!

Solution:

n + 5 <= c\*n

5 <= c\*n - n

5 <= n\*(c - 1)

5/(c - 1) <= n

Let’s choose the constants:

c = 2; n0 = 5

It holds for all n > n0, because the right side of the inequality can be just greater as we choose a greater n.

**Ex 2:**

Prove that 5n^4 +3n^3 +2n^2 +4n+1 is O(n4).

Proof: Note that 5n^4 +3n^3 +2n^2 +4n+1 ≤ (5+3+2+4+1)n^4 = cn^4, for c = 15, when n ≥ n0 = 1.

**Ex 3:**

(We rely on the mathematical fact that logn ≤ n for n ≥ 1):

Prove that 5n2 +3nlog n+2n+5 is O(n2).

Proof: 5n2 +3nlog n+2n+5 ≤ (5+3+2+5)n2 = cn2, for c = 15, when n ≥ n0 = 1.

**Ex 4:**

Prove that 20n3 +10nlog n+5 is O(n3).

Proof: 20n3 +10nlog n+5 ≤ 35n3, for n ≥ 1.

**Ex 5:**

Prove that 3log n+2 is O(logn).

Proof: 3logn+ 2 ≤ 5log n, for n ≥ 2. Note that logn is zero for n = 1. That is why we use n ≥ n0 = 2 in this case.

**Ex 6:**

Prove that 2(n+2) is O(2n).

Proof: 2n+2 = 2n ·22 = 4·2n; hence, we can take c = 4 and n0 = 1 in this case.

**Ex 7:**

Prove that 2n+100log n is O(n).

Proof: 2n+100log n ≤ 102n, for n ≥ n0 = 1; hence, we can take c = 102 in this case.

**Big-Omega:**

**Ex 1:**

Prove that 3n log n − 2n is Ω(n log n).

Proof: 3n log n − 2n = n log n + 2n(log n − 1) ≥ n log n for n ≥ 2; hence, we can take c = 1 and n0 = 2 in this case.

**Big-Theta:**

**Ex 1:**

Prove that 3n log n + 4n + 5 logn is Θ(n log n).

Proof: 3n log n ≤ 3n log n + 4n + 5 logn ≤ (3+4+5) n logn for n ≥ 2.

**Csak gyakorlas keppen:**

**Ex 1:**

Prove that 3n log n + 4n + 5 logn is O(n log n).

Proof: 3n log n + 4n + 5 logn < 3n log n + 4n log n + 5n log n = 12n log n. We can take c=12 and n0=2.

**Ex 2:**

Prove that 3n log n - 4n + 5 logn is O(n log n).

Proof: 3n log n - 4n + 5 logn < 3n log n + 5 logn < 3n log n + 5n logn = 8n logn. We can take c=8 and n0=2.

Check: f(n)<= cg(n).

6 log 2 - 8 + 5 log 2 <= 16 log 2

6 - 8 + 5 <= 16

3 <= 16.

**Limits**

**Ex 1:**

*f*(*n*) = 2*n* + *√*3*n* + 2

Describe the asymptotic behavior of f (n) using Theta-notation.

Justify your answer.

Solution:

f(n) is Theta(n).

Proof:

lim (2*n* + *√*3*n* + 2 )/n = lim 2n/n + lim ((3n)^1/2)/n + lim 2/n = 2.

Limit calculator:

<https://www.wolframalpha.com/calculators/limit-calculator>